FREELY CONVECTIVE HEAT TRANSFER AT THE EXTERNAL SURFACE OF A VERTICAL ISOTHERMAL CYLINDER

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Numerical and experimental results on the freely convective heat transfer of a vertical isothermal cylinder are analyzed. Formulas are proposed for the calculation of local and average heat-transfer coefficients for Pr = 0.01-100.

In practical heat calculations, the heat transfer at the external surface of a vertical cylinder in a large motionless volume is determined from data for a vertical plane wall [1, 2]. However, such results are only valid for cylinders of large diameter. When high accuracy of the heat-transfer calculation is required (for example, in the construction of precision measuring instruments), there is a need for formulas taking into account the effect of transverse curvature on the heat transfer. It may be noted that there are empirical formulas for the limiting case of thin heated wires.

Calculations of the heat transfer at a vertical cylinder are based on the equation of a freely convective axisymmetric boundary layer [3]:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+g\beta(T-T_\infty),$$
$$\frac{\partial ru}{\partial x}+\frac{\partial rv}{\partial r}=0, \tag{1}$$
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right),$$

with boundary conditions

$$u=0,\ v=0,\ T=T_w \text{ for } r=R;$$
$$u=0,\ T=T_\infty \text{ as } r\to\infty. \tag{2}$$

Equation (1) cannot be used for heat-transfer calculations in the case of thin vertical wires when the boundary-layer thickness exceeds the cylinder diameter [4].

Using the self-consistent variables ξ and η for the stream function $\psi(x, y)$ and the excess temperature [5],

$$\psi=4\nu R\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}F(\xi,\eta),\quad \theta=\frac{T-T_\infty}{T_w-T_\infty},$$
$$\xi=\frac{2x}{R}\left(\frac{\mathrm{Gr}_x}{4}\right)^{-1/4},\quad \eta=\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}\frac{r^2-R^2}{2Rx}, \tag{3}$$

system of equations (1) with the boundary conditions (2) may be rewritten as follows:

$$(1+\xi\eta)\frac{\partial^3F}{\partial\eta^3}+3F\frac{\partial^2F}{\partial\eta^2}-2\left(\frac{\partial F}{\partial\eta}\right)^2+\theta+\xi\frac{\partial^2F}{\partial\eta^2}=\xi\left(\frac{\partial F}{\partial\eta}\cdot\frac{\partial^2F}{\partial\eta\partial\xi}-\frac{\partial F}{\partial\xi}\frac{\partial^2F}{\partial\eta^2}\right), \tag{4}$$
$$\frac{1}{\mathrm{Pr}}(1+\xi\eta)\frac{\partial^2\theta}{\partial\eta^2}+3F\frac{\partial\theta}{\partial\eta}+\frac{1}{\mathrm{Pr}}\xi\frac{\partial\theta}{\partial\eta}=\xi\left(\frac{\partial F}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial F}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right),$$

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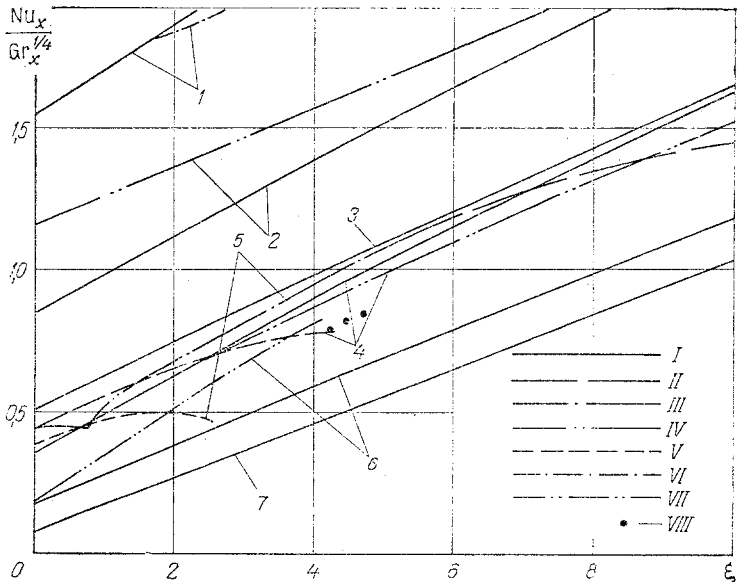

Fig. 1. Local heat-transfer coefficient versus curvature parameter ξ: 1) Pr = 100; 2) 10; 3) 2.0; 4) 0.72; 5) 0.733; 6) 0.1; 7) 0.01; I) accurate solution; II) [8]; III) [12]; IV) [9]; V) [7]; VI) [6]; VII) method of local self-consistency; VIII) experimental [8].

$$F=0,\ \frac{\partial F}{\partial\eta}=0,\ \theta=1 \ \text{for}\ \eta=0;\ \frac{\partial F}{\partial\eta}=0,\ \theta=0\ \text{as}\ \eta\to\infty. \tag{5}$$

The literature contains a large number of solutions of Eqs. (1)-(2) by the perturbation method [5-7], the integral method [8, 9], the method of local similarity [10], a numerical method [11], and a method close to the self-consistent method [12]. All the calculations were made for a narrow range of the parameters and were not generalized.

The system of equations (4) with the boundary conditions (5) is considered numerically by a finite-difference method for Pr = 0.01-100 and values of the longitudinal coordinate up to ξ = 10. In the initial cross section (ξ = 0) the result transforms to the solution of the known self-consistent problem of free convection at a vertical isothermal plane plate [1-5]. The results of numerical calculation for a plate (collected in [3]) may be approximated by the relation

$$\left(\frac{Nu_x}{Gr_x^{1/4}}\right)_{pl}=0.40\,Pr^{0.3} \tag{6}$$

with error 5% for 0.7 < Pr < 100 and 10% for Pr = 0.1. For Pr > 100 ($Nu_x \sim Pr^{1/4}$) and Pr < 0.1 ($Nu_x \sim Pr^{1/2}$), Eq. (6) cannot be used, as is evident from the asymptotic solutions.

In Fig. 1 numerical results are shown for the local heat transfer at a vertical cylinder and, for comparison, other theoretical and experimental data are given. Note that for Pr = 0.7 and ξ = 1, the curvature leads to a 20% increase in the heat transfer in comparison with a plane plate, and the correction factor rises with further increase in ξ. Using Fig. 1, it is possible to estimate the limits of application of approximate methods for the heat-transfer calculation. The agreement with the numerical calculations of [11] and the local similarity method of [10] is the most accurate, although not for all values of Pr.

The curves of the numerical calculation may be approximated by the relations

$$\frac{Nu_x}{Gr_x^{1/4}}=\left(\frac{Nu_x}{Gr_x^{1/4}}\right)_{pl}+0.12\,\xi=0.40\,Pr^{0/3}+0.12\,\xi, \tag{7}$$

$$Nu_x=(Nu_x)_{pl}+0.34\,\frac{x}{R} \tag{8}$$

with accuracy 5% for Pr = 0.7-100 and $0\leqslant\xi\leqslant 10$. For Pr = 0.1 and 0.01 and ξ = 5, the error is 13 and 19%, respectively. An expression of analogous structure is proposed for $\xi\leqslant 0.79\cdot Pr^{0.3}$ in [6]:

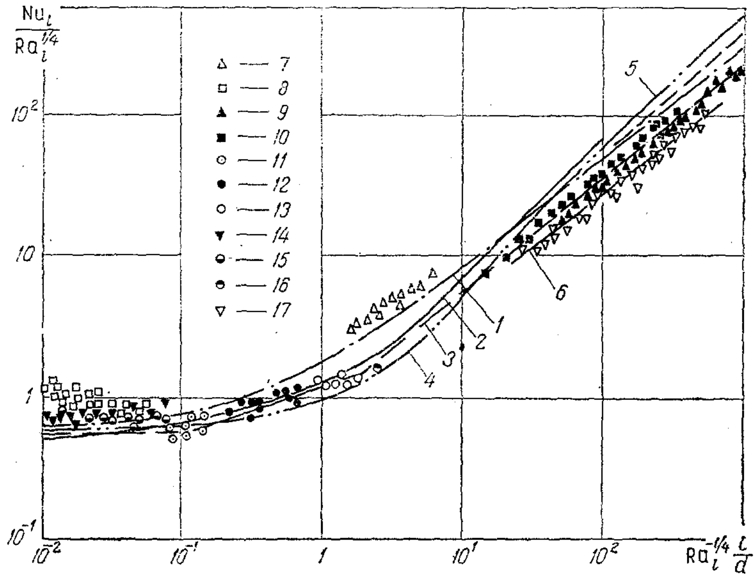

Fig. 2. Comparison of experimental and theoretical data on mean heat-transfer coefficient. Theoretical relations: 1) Eq. (12) [13-15]; 2) Eq. (10); 3) Eqs. (16)-(18) [12]; 4) Eq. (13), Pr = 0.72 [9]; 5) Eq. (14) [16, 17]; 6) Eq. (15) [19, 20]. Experimental data: 7) water, $d/l = 2 \cdot 10^{-2}$ [3]; 8) air, d/l = 1-4 [3, 24]; 9) air and helium, $d/l = 1.6 \cdot 10^{-4}$ [20]; 10) air and helium, $d/l = 6.4 \cdot 10^{-4}$ [20]; 11) water, $d/l = 4 \cdot 10^{-2}$ [18]; 12) water, $d/l = (4-5) \cdot 10^{-3}$ [18]; 13) water, $d/l = 2 \cdot 10^{-3}$ [18]; 14) air, ethylene glycol, $d/l = 8.5 \cdot 10^{-2}-5 \cdot 10^{-1}$ [24]; 15) water, oil, $d/l = 8 \cdot 10^{-2}$ [21-23]; 16) air [8]; 17) helium, argon, $d/l = 1.9 \cdot 10^{-4}$ [19].

$$\mathrm{Nu}_x = (\mathrm{Nu}_x)_{\mathrm{pl}} + 0.435\,\frac{x}{R} \tag{9}$$

with the same limits of accuracy as Eq. (8). It is a feature of Eqs. (8) and (9) that the second term, determining the correction for transverse curvature, is independent of Pr and proportional to the longitudinal coordinate x.

The only experiment on the local heat transfer of a cylinder in air [8] cannot serve as the basis for verifying the theoretical relations.

Experimental investigations have been made for average values of the heat-transfer coefficient. However, the main experiments correspond to large (short cylinder) and small (thin wire) values of d/l (Fig. 2). Note the wide ranges of the main parameters: Pr (air, helium, water, oil, ethylene glycol, and attenuated gas); $Ra_l = 10^3-10^9$; $d/l = 1.6 \cdot 10^{-4}-4$ [16-24].

The mean heat-transfer coefficient is obtained by integrating the local value over the height:

$$\frac{\mathrm{Nu}_l}{\mathrm{Ra}_l^{1/4}} = 0.53\,\mathrm{Pr}^{0.05} + 0.68\left(\mathrm{Ra}_l^{-1/4}\,\frac{l}{d}\right), \tag{10}$$

$$\overline{\mathrm{Nu}}_l = (\overline{\mathrm{Nu}}_l)_{\mathrm{pl}} + 0.68\,\frac{l}{d},\quad \frac{l}{d}\,\mathrm{Gr}_l^{-1/4} < 2. \tag{11}$$

Extrapolation of Eq. (10) to large values of $Ra_l^{-1/4}(l/d)$ gives values of the heat transfer that are too high (Fig. 2). This indicates that such calculations may be used for a cylinder but not for wires.

Consider other formulas obtained by approximate methods or by analysis of experimental data. In [13-15] the hypothesis of a steady film was used to derive the relation

$$\frac{\overline{\mathrm{Nu}}_l}{\mathrm{Ra}_l^{1/4}}\exp\left[-2\,\mathrm{Ra}_l^{1/4}\,\frac{l}{d}\Big/\left(\frac{\mathrm{Nu}_l}{\mathrm{Ra}_l^{1/4}}\right)\right] = 0.6. \tag{12}$$

By an integral method, a linear relation in terms of the curvature was obtained for the heat transfer [9]:

$$\frac{\overline{\mathrm{Nu}}_l}{\mathrm{Ra}_l^{1/4}}=\frac{3}{4}\left(\frac{7\mathrm{Pr}}{5(20+21\,\mathrm{Pr})}\right)^{1/4}+\frac{4(272+315\,\mathrm{Pr})}{35(64+63\,\mathrm{Pr})}\left(\mathrm{Ra}_l^{-1/4}\frac{l}{d}\right). \tag{13}$$

Analysis of experimental data in [16, 17] led to the recommendation of the following expression for thin shells:

$$\frac{\overline{\mathrm{Nu}}_l}{\mathrm{Ra}_l^{1/4}}=0.45\,\mathrm{Ra}_l^{-1/4}\frac{l}{d},\quad \mathrm{Ra}_l^{-1/4}\frac{l}{d}>20. \tag{14}$$

The relation proposed for an attenuated gas in [19, 20] was

$$\frac{\overline{\mathrm{Nu}}_l}{\mathrm{Ra}_l^{1/4}}=\frac{2\mathrm{Ra}_l^{-1/4}\dfrac{l}{d}}{\ln\left(1+4.47\mathrm{Ra}_l^{-1/4}\dfrac{l}{d}\right)}, \tag{15}$$

which is in good agreement with experiment in the range $10^{1.1} < \mathrm{Ra}_l^{-1/4}(l/d) < 10^{2.7}$. In [12], three regions were distinguished:

$$\frac{\overline{\mathrm{Nu}}_l}{\mathrm{Ra}_l^{1/4}}=0.57,\quad \mathrm{Ra}_l^{-1/4}\frac{l}{d}<0.1, \tag{16}$$

$$\frac{\overline{\mathrm{Nu}}_l}{\mathrm{Ra}_l^{1/4}}=1.3\left(\mathrm{Ra}_l^{-1/4}\frac{l}{d}\right)^{0.36},\quad 0.1<\mathrm{Ra}_l^{-1/4}\frac{l}{d}<2.1, \tag{17}$$

$$\frac{\overline{\mathrm{Nu}}_l}{\mathrm{Ra}_l^{1/4}}=0.87\left(\mathrm{Ra}_l^{-1/4}\frac{l}{d}\right)^{0.87},\quad \mathrm{Ra}_l^{-1/4}\frac{l}{d}>2.1, \tag{18}$$

which correspond, in the authors' classification, to short cylinders (plane plates), long cylinders, and wires. Experimental confirmation of Eqs. (16)-(18) for four values of Pr is given in [18].

In [25] an upper limit at which, with 5% error, the effect of transverse curvature on the heat transfer may be neglected is suggested. This limit is $\mathrm{Ra}_l^{-1/4}(l/d) < 0.033$ according to approximate calculations, but the accurate value is 0.039.

Thus, for $\mathrm{Ra}_l^{-1/4}(l/d) < 2$ the heat transfer of short and long cylinders may be calculated using Eq. (10), which is derived from numerical data. For thin wires [$\mathrm{Ra}_l^{-1/4}(l/d) > 2$], Eq. (18) gives the best agreement with experiment. Using Eqs. (10)-(15) for wires leads to values of the heat transfer that are too high.

NOTATION

x, r, longitudinal and radial coordinates; u, v, projections of velocity vector on x and r axes; T, temperature; ν, kinematic viscosity; a, thermal diffusivity; R, d, radius and diameter of cylinder; ψ, stream function; F, dimensionless stream function; θ, dimensionless excess temperature; ξ, η, self-consistent variables of longitudinal and transverse coordinates; $\mathrm{Nu}_x = \alpha_x x/\lambda$, Nusselt number; $\mathrm{Pr} = \nu/a$, Prandtl number; $\mathrm{Gr}_x = q\beta(T_w - T_\infty)x^3/\nu^2$, Grashof number; $\mathrm{Ra} = \mathrm{GrPr}$, Rayleigh number. Indices: ∞, external flow; w, wall; pl, plate; x, local value; l, value averaged over the length l.

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RHEODYNAMICS OF NONLINEARLY VISCOPLASTIC LIQUIDS

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Experimental results are given for the flow of an anomalous liquid and the possibility of a description of the flow curve that is invariant with respect to the geometry of the transporting medium is discussed.

There has recently been a sharp increase in the number of experimental and theoretical works devoted to the flow of non-Newtonian liquids. Of particular interest, because of their wide distribution, is the case of viscoplastic liquids, which are characterized by anomalous viscosity and also plastic properties such that, beyond some limiting shear stress, the liquid begins to flow.

The rheodynamic study of such systems is also of interest, since it affords the possibility of improving the technological processes in polymer reprocessing, in the production of composite materials and paint and varnish coatings, in petroleum reprocessing and elsewhere in the petroleum industry, and so on.

The extensive experimental material that has been accumulated on the flow of viscoplastic liquids in transporting media of various geometries [1, 2] indicates that the dependence of the tangential stress τ on the shear-rate gradient $\dot{\gamma}$ is nonlinear. For the solution of specific problems, the flow curves are approximated by one of the rheological models (Bingham—Shvedov, Balkley—Herschel, Caisson, etc.); note that the rheological parameters of the

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